Fluctuations and Decoherence in Chaos-Assisted Tunneling

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We study quantum dynamical tunneling between two symmetry-related islands of stability in the phase space of a classically chaotic system. The setting for these experiments is the motion of carefully prepared samples of cesium atoms in an amplitude-modulated standing wave of light. We examine the dependence of the tunneling dynamics on the system parameters and indicate how the observed features provide evidence for chaos-assisted (three-state) tunneling. We also observe the influence of a noisy perturbation of the standing-wave intensity, which destroys the tunneling oscillations, and we show that the tunneling is more sensitive to the noise for a smaller value of the effective Planck constant.

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Much progress has been made in the study of the quantum dynamics of classically chaotic systems by exploring how quantum behavior is influenced by structures in the classical phase space. A particularly challenging regime of study is that of mixed systems, where regular and chaotic regions coexist in the same phase space, due to the complexity of the dynamics. One manifestly quantum effect that has come to light in these systems is dynamical tunneling [1], where a quantum wave packet can oscillate coherently between two symmetry-related regular regions in phase space, even though the classical transport between these regions is forbidden. This effect specifically refers to the situation in which the classical transport is forbidden by the dynamics of the system and not by a potential barrier, as is the case in the more familiar barrier tunneling. Although markedly different from barrier tunneling, dynamical tunneling proceeds along lines similar to barrier tunneling in the symmetric double-well potential [2], in that the tunneling occurs as a pair of nearly degenerate states ("quasidoublet") with opposite parity dephase. Subsequently, it was found that the chaotic regions could also have a substantial effect on the tunneling rate between the regular regions [3,4], an effect that has since become known as chaos-assisted tunneling [2,5].

In this Letter, we will focus on the original three-level model of chaos-assisted tunneling [2,5] (see also [6–8] for particularly lucid discussions of this model). This model, also referred to as a singlet-doublet crossing [7], involves the interaction of a tunneling quasidoublet of states localized on the symmetry-related regular regions (and having vanishingly small splitting) with a third state associated with the chaotic region. These states can be energy eigenstates in autonomous systems or Floquet (quasienergy) states in the case of time-periodic systems, as we consider in the experiment. We initially consider these three states for a parameter regime where the singlet and the doublet are separated in (quasi)energy and thus do not interact. As a system parameter varies, the chaotic state can "collide" with the doublet, producing a three-state avoided crossing. Because the two regular states have opposite parity and the chaotic state also has definite parity, the chaotic state interacts only with one member of the doublet. If the interaction is sufficiently strong, the splittings can be large at the avoided crossing, leading to an enhanced tunneling rate. The chaotic states fluctuate erratically as system parameters vary, leading to irregular fluctuations in the tunneling rate over orders of magnitude [2,6].

Chaos-assisted tunneling has only recently become amenable to experimental study. The first evidence for this effect came from a spectroscopic study of the modes of a microwave resonator, where the authors demonstrated an enhancement in the quasidoublet splitting for states near the border between the stable and the chaotic regions [9]. More recently, we have reported the observation of chaos-assisted dynamical tunneling of ultracold atoms between two islands of stability in a modulated optical lattice [10]. In this work, we demonstrated the enhancement of the tunneling rate relative to integrable tunneling in the unmodulated lattice (Bragg scattering), and we also established the existence of an oscillation of the atomic population in the chaotic region, which pointed to the role of the chaotic region in mediating the tunneling. (A related experiment [11] studied dynamical tunneling of a Bose condensate between another pair of islands in a modulated optical lattice.) The goal of the present Letter is to report the first observation of several important features of chaos-assisted/three-level tunneling, including multiple oscillations due to dephasing of different pairs of the three levels. We also study the dependence of the tunneling rate on a system parameter, for which tunneling appears only in a certain range, and which shows a dependence that is not expected for ordinary (two-state) dynamical tunneling. Finally, we study the effects of noise on tunneling, demonstrating both the suppression of tunneling and an increased sensitivity to noise for a larger action scale of the system.

The experimental apparatus and procedure used here have been described previously [10]. We study the dynamics of cold cesium atoms exposed to an amplitude-modulated standing wave of far-detuned laser light, which provides a nearly conservative spatial potential

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for the atomic motion. In scaled units, the Hamiltonian for the center-of-mass atomic motion is $H = p^2/2 - 2\alpha \cos^2(\pi t) \cos(x)$ [10], where α is proportional to the laser intensity and the inverse of the detuning to the nearest resonance (50 GHz for the experiments here). The other parameter in these scaled units is the effective Planck constant $\bar{k} := 8\omega_r T$, where $\omega_r/2\pi = 2.07$ kHz for the cesium D_2 transition, and T is the period of the modulation. Because the scaled coordinate operators satisfy $[x, p] = i\bar{k}$, this parameter $\bar{k}$ controls the size of the atomic wave packet in the classical phase space. The phase space for this system is characterized by three primary resonances [6], corresponding to the three (complex) frequency components of the modulation. We study the tunneling of the atoms between the two symmetry-related resonances that move with opposite velocities; the third, stationary resonance does not directly participate in the tunneling.

A crucial aspect in successfully observing tunneling oscillations is the procedure for quantum-state preparation, as described previously [10]. After the atoms are collected in a standard magneto-optic trap and further cooled in a three-dimensional optical lattice, a subset of atoms with a narrow momentum distribution (having a half width at half maximum of $0.03 \times 2\hbar k_L$) about $p = 0$ is selected using velocity-selective, stimulated Raman transitions. The far-detuned standing wave, whose intensity is controlled by an acousto-optic modulator (AOM), is then turned on adiabatically, so that the atoms localize in the potential wells. After a sudden shift of the phase of the standing wave and a delay of appropriate duration, the atoms are localized in phase space (in nearly a minimum-uncertainty configuration) and centered on one of the symmetry-related islands of stability. The subrecoil velocity selection and the two-photon nature of the momentum transfer from the optical lattice to the atoms ensure that only states with momentum nearly an integer multiple of $2\hbar k_L$ are populated, which, along with being localized on a resonance, is a necessary condition for tunneling to occur [6,10]. For the data presented here with $\bar{k} = 2.08$ ($T = 20\ \mu$s), the initial wave-packet distribution (in momentum) was peaked at $4.2 \times 2\hbar k_L$, with a width $\sigma_p = 1.7 \times 2\hbar k_L$, and $8.2 \times 2\hbar k_L$, with a width $\sigma_p = 2.1 \times 2\hbar k_L$, for the data with $\bar{k} = 1.04$ ($T = 10\ \mu$s). The potential amplitude α was determined by measuring the small-amplitude oscillations of the atoms in a deep, stationary standing wave, and comparing the data directly to quantum simulations; the values quoted here have a 5% uncertainty.

As mentioned above, one signature of chaos-assisted (three-level) tunneling is a nonuniversal dependence of the tunneling rate on the system parameters. An experimental measurement of the tunneling rate as a function of α, with $\bar{k}$ fixed at 2.08, is shown in Fig. 1. Each tunneling rate was extracted from an average of ten measurements of $\langle p \rangle$ vs t, of which two examples are shown in Fig. 2. Tunneling is visible in the range of α from about 7 to 14, but is suppressed outside this range. Below this range, the tunneling is presumably too slow to be observed, and above this range the outer islands have completely dissolved into the chaotic sea, so that we no longer expect clean tunneling to occur. There are two particularly interesting features to notice in the variation of the tunneling rate. The first is that the tunneling rate *decreases* as a function of the coupling strength α. This dependence is the opposite of our expectation of direct (two-state) tunneling, where the tunneling rate should increase with α (for Bragg scattering, an analogous, direct-tunneling process in an unmodulated standing wave, the tunneling rate increases with a power-law dependence on α [12]). This behavior is thus strong evidence that the tunneling is chaos-assisted, where one or more chaotic levels have a definite influence on the doublet splitting.

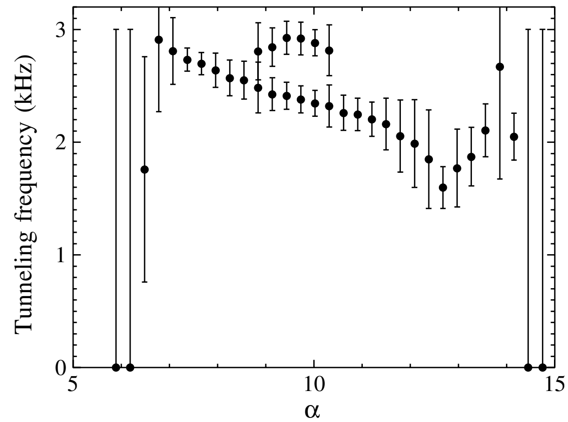

FIG. 1. Dependence of the tunneling rate on the well depth α, for $\bar{k} = 2.08$ ($T = 20\ \mu$s). The periods were extracted from measurements of $\langle p \rangle$ vs t using both numerical Fourier transform and nonlinear fitting techniques. In the range of α from 8.9 to 10.3, two distinct frequencies can be resolved in the tunneling data. The zero-frequency data points at the edges of the plot indicate that no tunneling frequency can be extracted from the data at these locations.

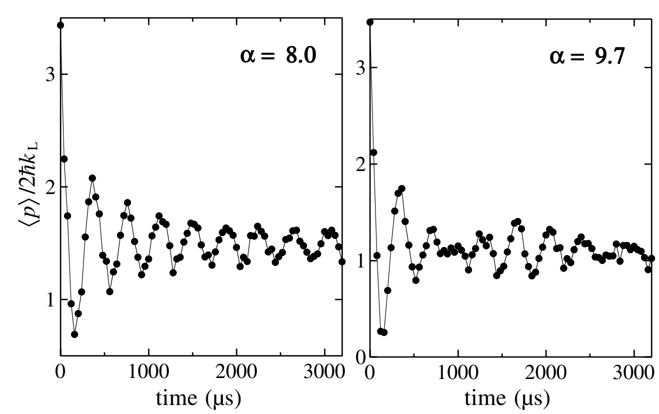

FIG. 2. Examples of tunneling oscillations for $\alpha = 8.0$ and $\alpha = 9.7$, corresponding to two of the measurements in Fig. 1. In the former case, a single frequency persists for the maximum duration of the optical-lattice interaction, while in the latter case the beating of two tunneling frequencies is apparent. The data points are connected by lines for clarity.

The second feature to notice is that two frequencies are clearly resolvable in a comparatively narrow window in α (from about 8.5 to 10.5). The one- and two-frequency behaviors of the tunneling are illustrated in Fig. 2, where one tunneling frequency is evident for $\alpha = 8.0$, and the beating of two frequencies is clearly apparent for $\alpha = 9.7$. This behavior is also consistent with the three-state model near the center of a singlet-doublet crossing. In this model, the initial wave packet populates a regular state (localized on the islands) and two hybrid states, which have population in both the islands and in the chaotic sea (i.e., these are the two interacting states with the same parity). There should thus be two frequencies associated with the tunneling, corresponding to the two splittings between the regular state and the two hybrid states. In general, these two splittings will not be equal, but should be similar near the center of the avoided crossing, leading to two-frequency beating in the tunneling dynamics.

This two-frequency behavior accounts for the dephasing of two pairs of the quasienergy states in the three-level model, but the dephasing of the third pair (the two hybrid states) should also be visible in the experimental data. Because these are the two states that repel each other in the avoided crossing, this pair has a larger splitting than the other two pairs. Thus, we expect this pair of states to be responsible for a transport that is much faster than the island-tunneling process, involving the transfer of population between the regular island regions and the chaotic layer. This behavior is illustrated in the surface plot of Fig. 3, which is sampled 10 times per modulation period, in contrast to the other data in this Letter, which are

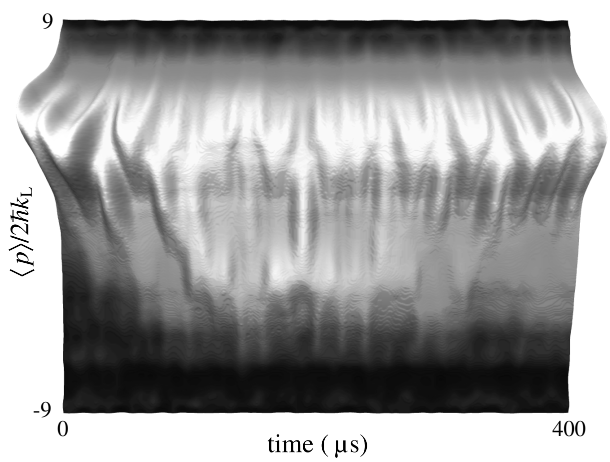

FIG. 3. Experimental momentum distribution evolution of chaos-assisted tunneling for $\bar{k} = 2.08$ ($T = 20\ \mu$s) and $\alpha = 7.7$. The distribution is sampled every 2 μs out to 400 μs, covering the first full tunneling oscillation. The classical island motion (with the same frequency as the modulation) is evident, as well as more complicated oscillations into the intermediate chaotic region near $p = 0$. The phase space is characterized by the two (symmetry-related) tunneling islands as well as a small doublet of stable islands near $p = 0$. These distributions are averaged over ten iterations of the experiment.

sampled at a single phase of the periodic modulation. This plot focuses on a single tunneling oscillation, which is the slowest process in this plot. Also visible is a fast complementary oscillation of the two island populations at the modulation frequency, which is a manifestation of the classical island motion [10]. A third, more interesting oscillation is also visible, where during several modulation cycles the atoms leave the islands and appear in the predominantly chaotic region between the islands. This oscillation is precisely what we expect from the dephasing of the two coupled states, and thus also strongly supports the three-level model in explaining the tunneling dynamics. Note, however, that although preliminary calculations [13], as well as the predominantly chaotic nature of the classical phase spaces studied here, suggest that the three-level dynamics are due to chaos-assisted tunneling, similar effects can be induced by states on regular regions in phase space [14]. We also note that more than three states may be important in understanding the present results, as suggested by the complicated nature of the oscillations in Fig. 3.

The tunneling that we have described relies on quantum coherence, and tunneling in classically chaotic systems is expected to be suppressed by dissipation [7,15], measurement [16], and noise [17]. Here we consider the effects of a noisy perturbation of the optical lattice by making the replacement $\alpha \to \alpha[1 + \varsigma(t)]$, where $\varsigma(t)$ is a randomly fluctuating quantity. This noise signal was generated by processing normally distributed random deviates using a digital Chebyshev low-pass filter (fourth order, with 0.1 dB passband ripple) before applying them to the AOM control signal. The cutoff frequency (0.5 MHz for the $\bar{k} = 2.08$, $T = 20\ \mu$s data, and 1 MHz for the $\bar{k} = 1.04$, $T = 10\ \mu$s data) was selected to be well within the 10 MHz modulation response of the AOM driver and to make the noise spectrum the same in scaled units for the different cases. The rms noise levels $\langle \varsigma^2(t) \rangle^{1/2}$ that we quote are the noise levels after the filter. Because the instantaneous noise level is proportional to the mean intensity, truncation effects due to noise deviations falling outside the dynamic range of the laser were rare except in the largest noise case (62% rms).

The response of the tunneling oscillations to the noise is illustrated in Fig. 4 for $\bar{k} = 2.08$ and $\bar{k} = 1.04$ ($\alpha = 11.2$ in both cases). As one might expect, the oscillations are destroyed as the noise level increases, causing damping of the oscillations on progressively shorter time scales. At the largest levels of noise, classical-like behavior (with noise) is recovered, in that the tunneling oscillations are suppressed. The noise also has the "direct" effect of causing relaxation to $p = 0$, because the noise permits transitions, both quantum and classical, out of the initial island of stability and into the chaotic sea. The more interesting feature about this data, though, is that, because the value of α is fixed between the two measurements and the tunneling periods are approximately the same (in scaled units),

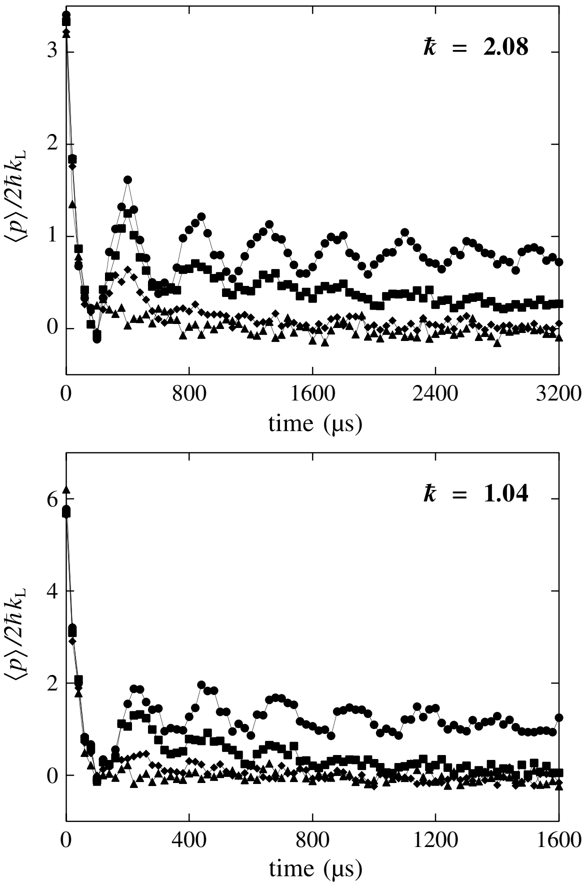

FIG. 4. Comparison of the effects of applied amplitude noise on the tunneling oscillations for $\alpha = 11.2$ and $\bar{k} = 2.08$ (upper graph) and $\bar{k} = 1.04$ (lower graph). The rms noise levels are 0% (circles), 15.7% (squares), 31% (diamonds), and 62% (triangles) in the $\bar{k} = 2.08$ case; and 0% (circles), 7.9% (squares), 15.7% (diamonds), and 31% (triangles) for the $\bar{k} = 1.04$ case. The tunneling is only completely suppressed at the 62% level in the first case, whereas in the second case the oscillations are already suppressed at the 31% level, and thus the tunneling is substantially more sensitive for the smaller value of $\bar{k}$. The data are averaged over ten realizations of noise.

we can compare the sensitivity of the system to the noise for two different values of $\bar{k}$. From the data, we see that the tunneling oscillations are suppressed at a much lower level of noise for the $\bar{k} = 1.04$ case than in the $\bar{k} = 2.08$ case (31% vs 62% rms). Recalling that $\bar{k}$ is the dimensionless Planck constant in scaled units, this comparison indicates that the tunneling in this system is more sensitive to decoherence as the system moves towards the classical limit (i.e., to a larger action scale compared to $\hbar$). This behavior is consistent with theoretical expectations because, for smaller $\bar{k}$, the phase-space structure in chaotic systems saturates on a smaller scale [18], thus being more easily influenced by decoherence (which causes diffusion in phase space). Related experimental results have demonstrated that Schrödinger-cat superposition states in the phase of a cavity field [19], in an atom interferometer [20,21], and in an ion trap [22,23] are more sensitive to decoherence when the separation of the components of the state increases (i.e., as the spacing of the interference fringes decreases). The present experimental results are of a fundamentally different nature, though: while these other experiments study the decoherence of a prepared superposition state, the interferences in the tunneling here are generated dynamically in this nonlinear system.

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