Floquet States in Chaos-Assisted Tunneling Experiments

Daniel A. Steck
Theoretical Division (T-8), MS B285, Los Alamos National Laboratory
Los Alamos, NM 87544

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Here are some calculated Floquet states for the experimental parameters in the paper: D. A. Steck, W. H. Oskay, and M. G. Raizen, "Fluctuations and Decoherence in Chaos-Assisted Tunneling."

First we consider the case of $\bar{k} = 2.077$, for which the Floquet spectrum is shown in Fig. 1. In this spectrum we can clearly see a doublet of states localized on the outer two islands of stability, where an avoided crossing with a third state with slightly lower quasienergy occurs around $\alpha = 7$. Already we can anticipate that this third state is associated with the chaotic region, because it has diverged from its partner of opposite parity already at around $\alpha = 1$ (regular states occur in quasidegenerate doublets).

Now we consider the distributions of the Floquet states in phase space to gain some more intuition for the dynamics. To do so, we plot Husimi distributions of these three states. However, this brings up some issues. First, how do we assess their relative importance in the tunneling dynamics? We know that they are important due to the structure of the spectrum, but we should also know how they overlap the island to see how much each state contributes. We could calculate the overlap with the experimental initial condition, but this is unsatisfactory for two reasons: first, we are after information that should be independent of any particular experimental realization, and second, the Gaussian distribution quoted in the paper is something of an idealization, as we expect distortion due to anharmonic evolution in the optical lattice during state preparation (thus requiring detailed modeling of the state-preparation process). The islands themselves set a natural center and aspect ratio for a Gaussian distribution, and thus we report the overlaps of the quasienergy states with these "natural Gaussians." As we will see, though, even this is unsatisfactory beyond the avoided crossing, where some of the states in the avoided crossing can have very small overlap with the island. This is expected behavior for an avoided crossing, and also suggests that more than three states are necessary to understand the tunneling dynamics for the larger α values in the experiment.

The second issue with Husimi distributions comes from their very definition. Since the Husimi distribution is a convolution of the Wigner distribution with a coherent state, the calculation of a Husimi distribution involves the rather arbitrary choice of the aspect ratio of the coherent state. The system here gives no preferred aspect ratio, except perhaps for the natural Gaussian mentioned above or the ground state of the unmodulated lattice; both of these choices lead to highly stretched states in phase space. The common choice is to simply set the aspect ratio to unity, but this is likewise arbitrary because it depends on the choice of unit scaling. Here I chose to follow the convention of Mouchet et al. "Chaos-assisted tunneling with cold atoms," *PRE* **64**, 016221 (2001), who set the aspect ratio to unity in units where the effective Planck constant is $\bar{k}/2\pi$. This choice of units puts the Hamiltonian in the form $H = p^2/2 + \gamma(1 + cos(x)cos(t))$, so that the modulation has the same frequency in space and time, and gives a nice middle ground for the spectrum of possible aspect ratios. With this choice the convolution kernel has the uncertainties $\Delta p = \sqrt{\pi\bar{k}}$ and $\Delta x = \sqrt{\bar{k}/4\pi}$. However, varying this choice of aspect ratio can give different results for some of the Husimi functions, as we will note below.

First consider the case $\alpha = 7.7, \bar{k} = 2.077$, for which the classical phase space and three Husimi functions are plotted in Fig. 2. This case is straightforward: one state is clearly localized just on the two islands, while the other two states (both with parity opposite to that of the first state) have population both in the islands and in the chaotic region. These two states do not populate the chaotic region in the same way, indicating either that other chaotic states are also important or that this value of α is not at the center of the crossing. Notice also how the second state (with a ring-shaped structure) neatly avoids the two center islands. These three states are the most populated by the natural Gaussian as well as by the experimental initial condition.

For the second case $\alpha = 9.7, \bar{k} = 2.077$ (Fig. 3), the states are mostly the same except for the state with quasienergy $\epsilon/2\pi\bar{k} = 0.39$. In this case it is not so clear that the state avoids the two center islands, although since these islands are so small, we could see this as some kind of "scarring" effect. However, this state is mostly spread through the chaotic region. Additionally, for convolution kernels that are more squeezed in p, more population appears on the outer two islands and less population appears near $p = 0$. This state is thus difficult to classify but appears to be essentially a chaotic state. These three states are still the three most populated by the natural Gaussian. However, for the experimental initial condition, the $\epsilon/2\pi\bar{k} = 0.39$ state is only the seventh most populated (with a scant 3% of the population), which indicates that the amount of population induced by a particular initial condition is not so valuable in deciding how much influence a state has on the dynamics. This is especially true in light of the complicated nature of the chaotic states, which makes their projection onto an initial state fairly sensitive to the details of the state.

For the last $\bar{k} = 2.077$ case considered here ($\alpha = 11.2$), shown in Fig. 4, the regular state is essentially the same but the other two states have evolved. The $\epsilon/2\pi\bar{k} = 0.36$ state has retreated towards the origin, possibly indicated scarring on the unstable fixed point, and the $\epsilon/2\pi\bar{k} = 0.26$ state also has more population at the origin. The latter state is only the fifth most populated state by the natural Gaussian, whereas for the experimental initial condition the $\epsilon/2\pi\bar{k} = 0.36$ state is the 11th most populated. Again all the caveats in the previous case apply, especially since this case is at the far edge of the avoided crossing and at the border where the outer islands dissolve.

Now we consider the "more classical" case $\bar{k} = 1.039$, first for the same value $\alpha = 11.2$ as in the previous case (Fig. 6). Here there are two states clearly localized on the two islands, while the third is localized (scarred) on the unstable fixed point. However, this third state is only the 18th most populated by the natural Gaussian state, which is again indicative of being past the avoided crossing, and that additional states are important. We can see this better by considering a value of $\alpha = 9.8$, closer to the center of the avoided crossing (Fig. 7), where these three states are again the three most populated states.

Hence, we can conclude by saying: the dynamics in the experiment certainly involve 3 (or more) levels, and appear to be chaos assisted; one must be careful when calculating and interpreting Husimi distributions; the degree of overlap of the experimental initial condition is not on its own a reliable indicator of the nature of the underlying dynamics.

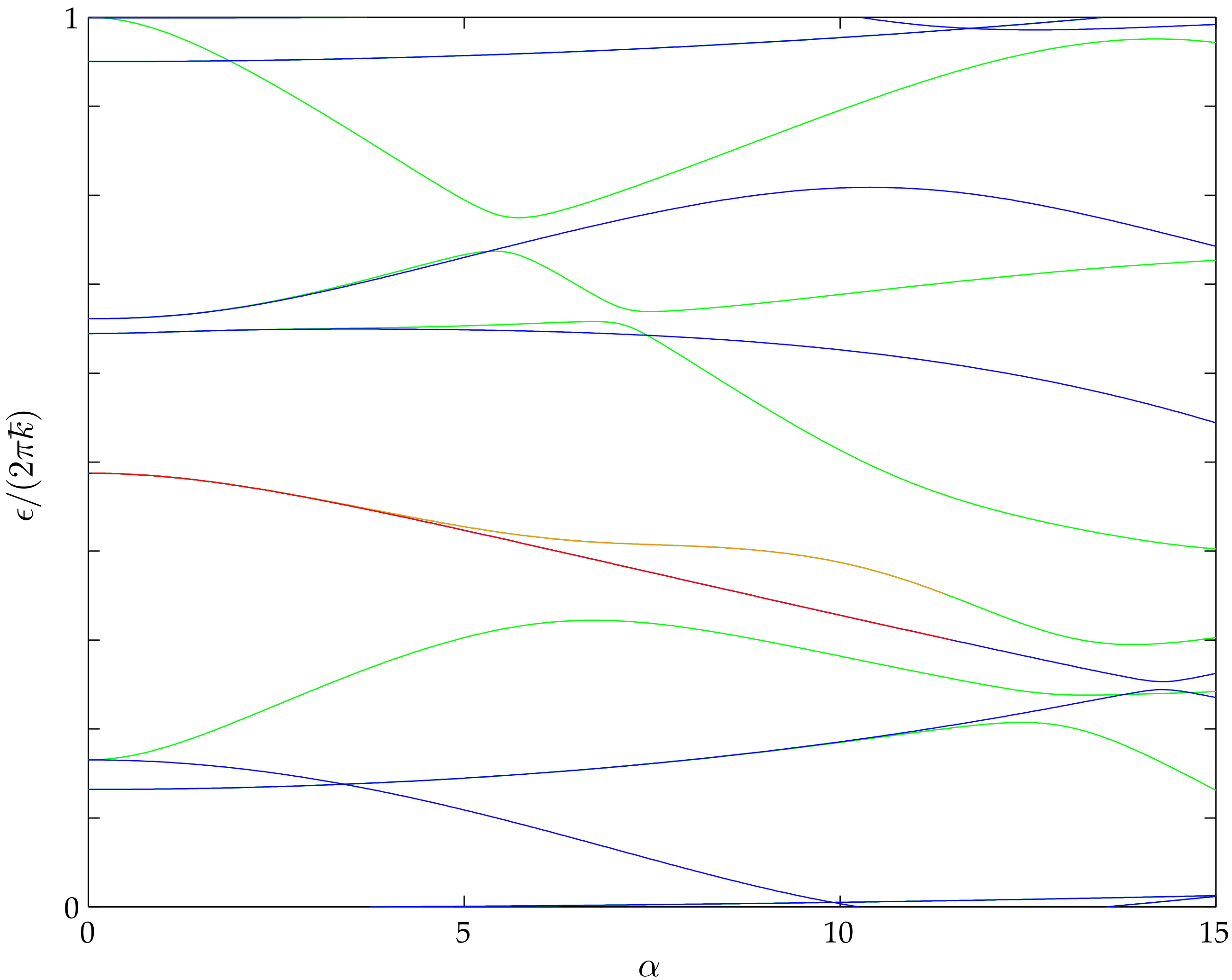

Figure 1: Floquet spectrum for $k\!\!\!{}^{-} = 2.077$. Even quasienergy states are green, odd are blue, and the even and odd states with maximal overlap with the island are shown in orange and red (see my dissertation for details).

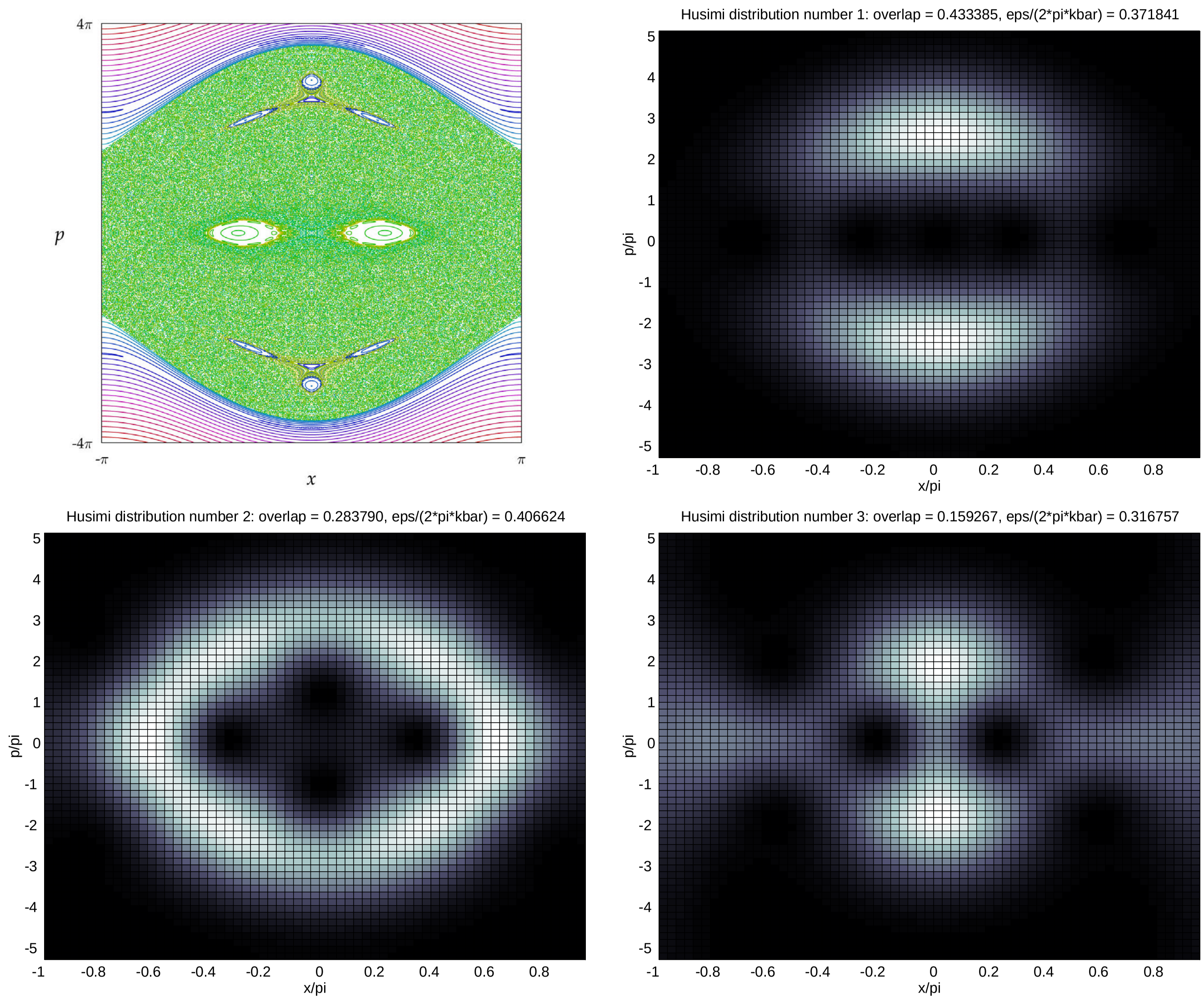

Figure 2: Classical phase space and three Husimi distributions for $\alpha = 7.7$, $\hbar = 2.077$.

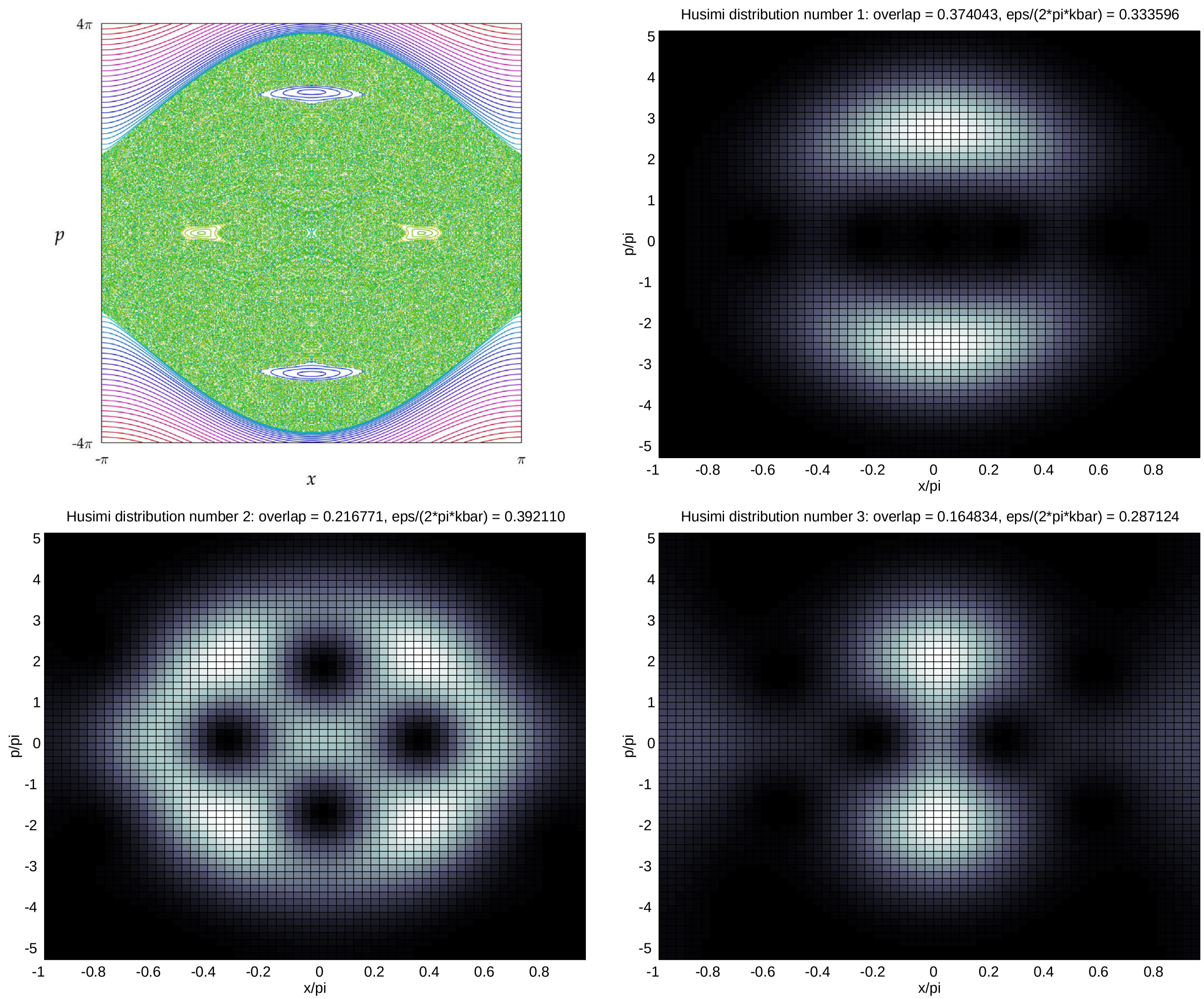

Figure 3: Classical phase space and three Husimi distributions for $\alpha = 9.7$, $\hbar = 2.077$.

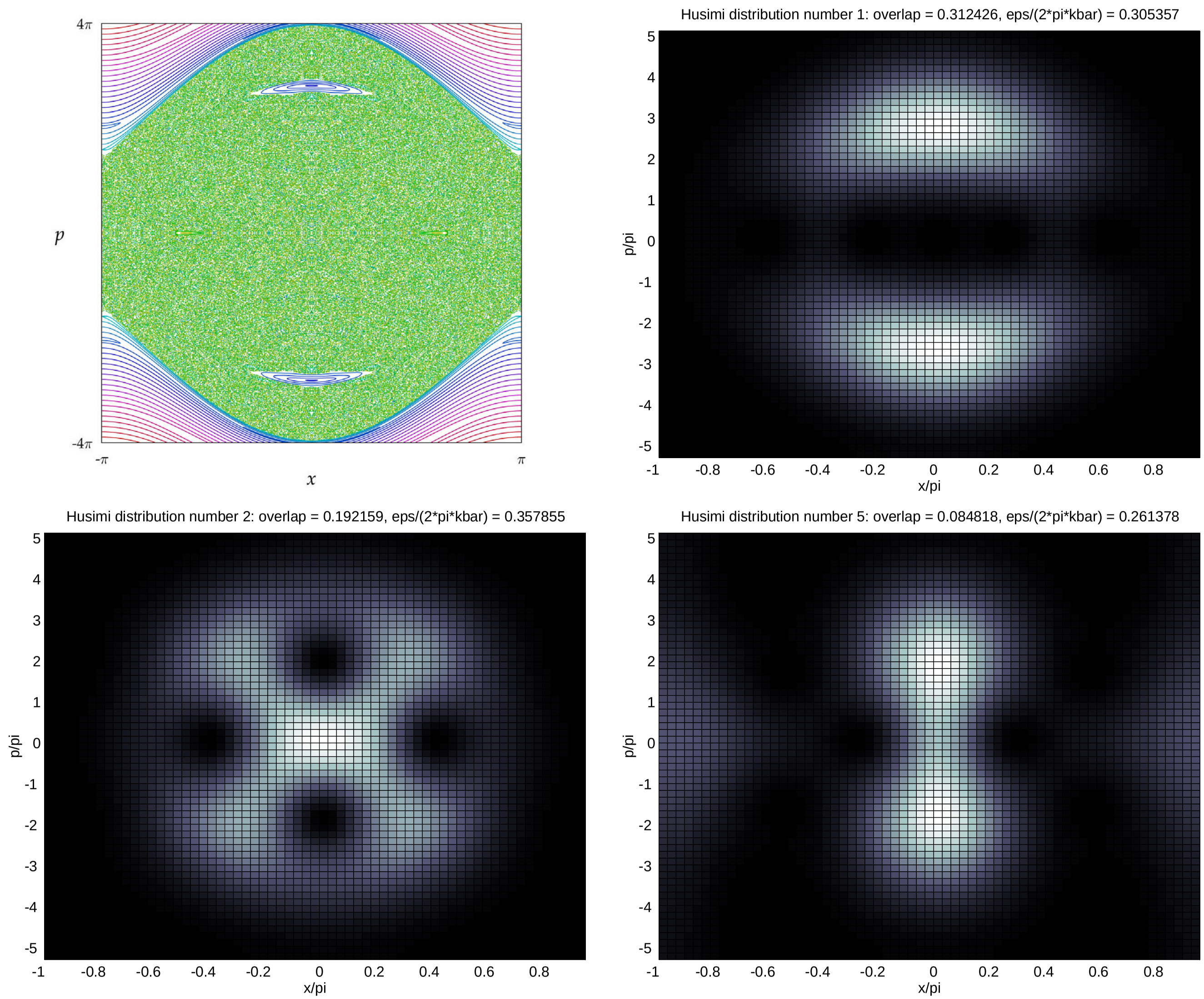

Figure 4: Classical phase space and three Husimi distributions for $\alpha = 11.2$, $\hbar = 2.077$.

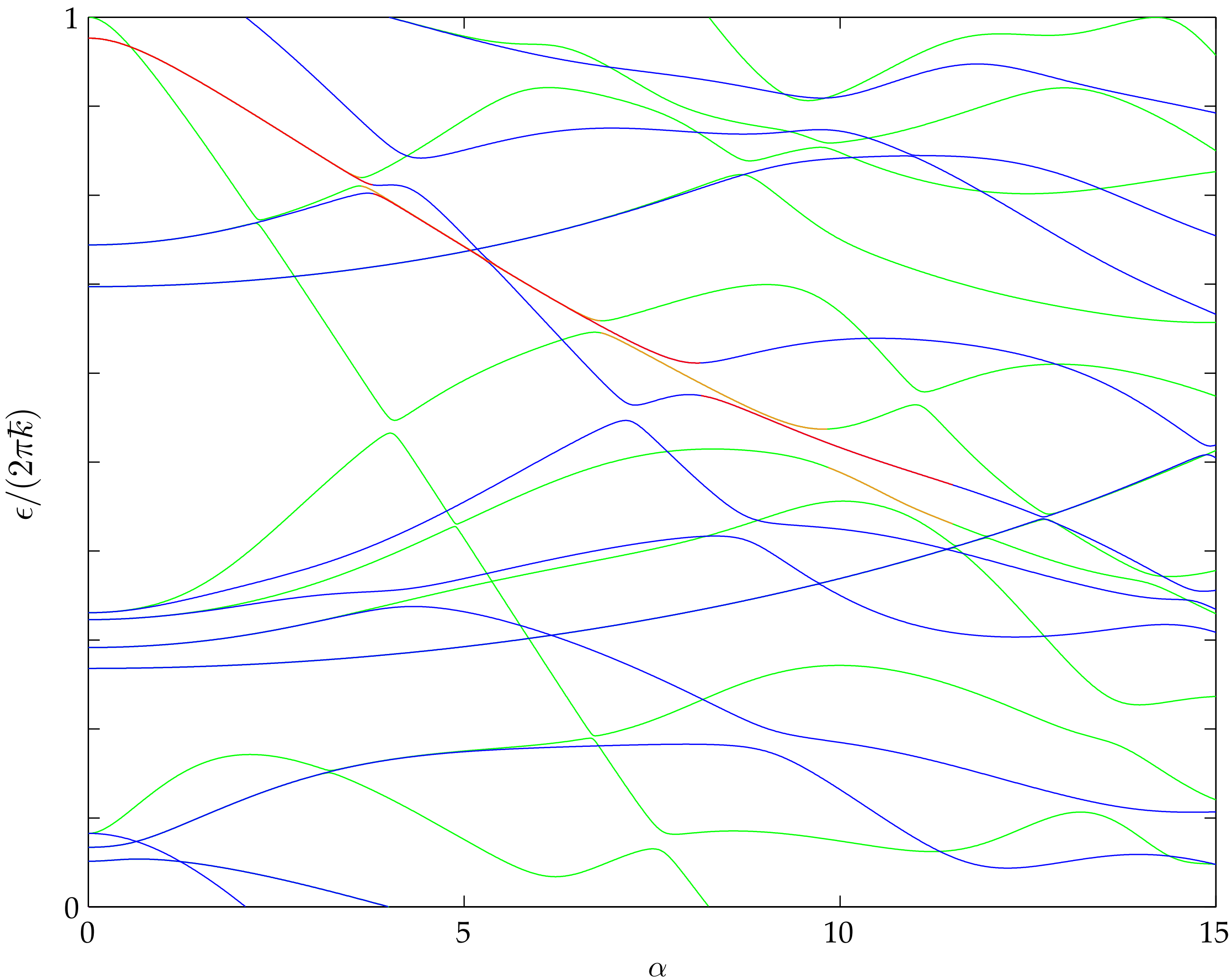

Figure 5: Floquet spectrum for $\hbar = 1.039$. Even quasienergy states are green, odd are blue, and the even and odd states with maximal overlap with the island are shown in orange and red (see my dissertation for details).

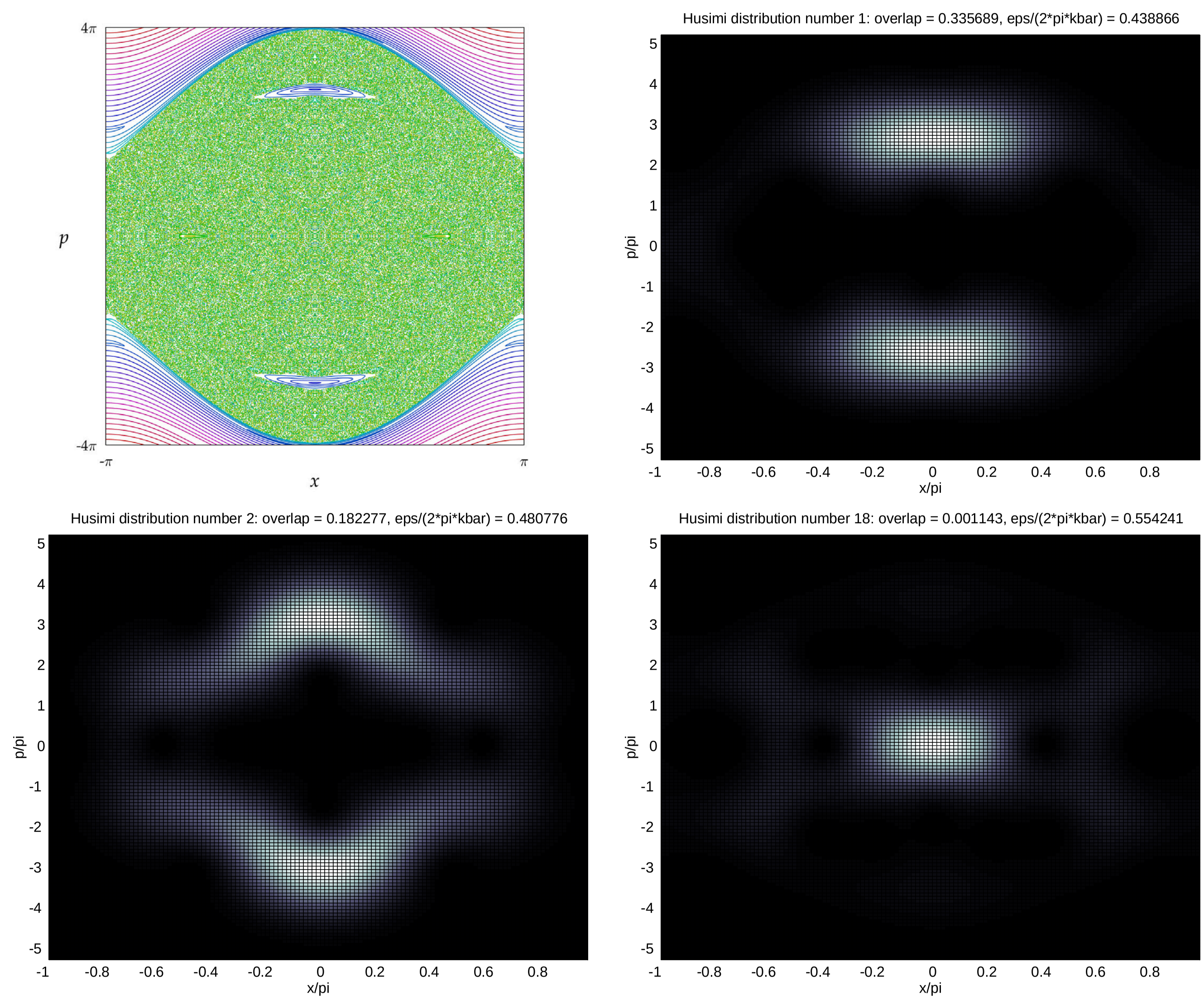

Figure 6: Classical phase space and three Husimi distributions for $\alpha = 11.2$, $\hbar = 1.039$.

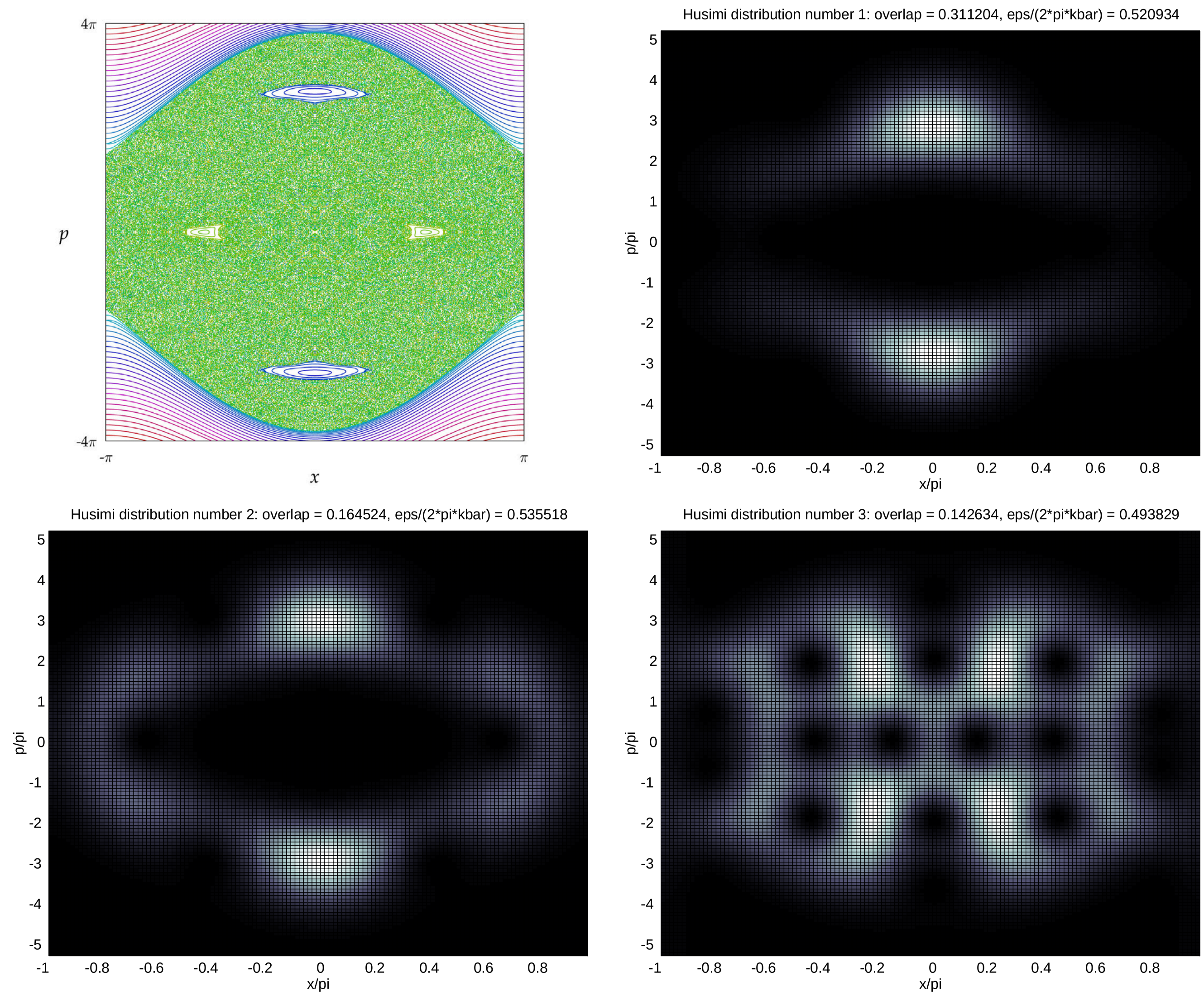

Figure 7: Classical phase space and three Husimi distributions for $\alpha = 9.8$, $\hbar = 1.039$.